Statistical Approximation of High-Dimensional Climate Models

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Abstract

We propose a general method for constructing low-dimensional approximations of complex dynamic models and apply the method to the climate model MAGICC to approximate the impact of CO_2 emissions on the global temperature. The standard practice is to construct plausible emission paths, compute the climate response they imply, and use statistical methods to estimate a simpler dynamical system. This approach is limited by the high cost of solving climate models and requires the construction of an efficient set of input scenarios. We show that a method using orthogonal polynomials is more efficient than the standard approach.

Keywords: Climate Change, Greenhouse Gas, Orthogonal Polynomials, Single Equation Models.

JEL codes: Q54, C20.

1 Introduction

Integrated Assessment Models (IAM) aim to merge dynamic models of the climate system with dynamic economic models to study their interactions and formulate policies related to limiting greenhouse gas emissions. It is currently intractable to merge the best climate models with modern macroeconomic models[1]. This problem is addressed by constructing low-dimensional dynamic systems that accurately represent the impact of world CO_2 emissions on world average global temperature. This is a valid reduction because CO_2 disperses rapidly in the atmosphere and the major impact of climate change is represented by average world temperature. The basic procedure for constructing the reduced model (often called the emulator) is to specify a set of emission paths, use each one as input into a complex climate model, observe the resulting temperature paths, and use the emissions and temperature data to specify an approximating dynamical system. The structure has a pooled cross-section nature since each path is a time series and multiple paths are used.

The statistical approach to constructing emulators has recently gained more support in the literature. As far as the physical nature of the climate system is concerned, statistical methods can produce models that adhere to basic climate physics. Kaufmann et al. (2013) examines two statistical models that link radiative forcing to global surface temperature as simulated by full complexity climate models (AOGCMs). They show that the statistical model that is supported more strongly by the data—the one with a stochastic trend potentially conveyed from radiative forcing to temperature—is consistent with the relation characterized by a globally averaged energy balance model. Pretis (2015) extends the analysis by establishing the relation between the cointegrated time series of three major climate variables and a two-component energy balance model.

Studies that estimate emulators of high-dimensional models generally rely on existing results of computer simulations (Young and Ratto, 2011; Castelletti et al., 2012). These data sets are designed to find the predictions of full complexity models in response to some commonly prescribed scenarios (e.g., IPCC-DDC 1998) and provide a consistent base for scientific studies such as inter-comparison projects. A widely-known example are the

[1]The climate models are PDEs with an initial value representing the physical state of the climate at some initial time. Myopic economic models (called "recursive" models in the IAM literature) are also initial value problems since information about the future does not affect current behavior in such models. Therefore, it is straightforward to merge myopic economic models with any climate model. Perfect foresight or rational expectations models are two-point boundary value problems with some states pinned down by initial conditions but other economic variables determined by transversality conditions at the terminal time. Merging even the smallest perfect foresight or rational expectations model with a large climate model would create an intractably large two-point boundary value problem.

Representative Concentration Pathways (RCPs) adopted in the most recent IPCC Assessment Report (van Vuuren et al. (2011)).

In cases when additional simulations are not computationally expensive, researchers generate a large collection of emission paths and the resulting temperature paths. For example, to emulate a climate model of intermediate complexity, Holden and Edwards (2010) construct an ensemble of possible future concentration profiles using the Latin hypercube method, and apply dimensionality reduction techniques to construct an emulator.

Unfortunately, the most detailed and complex climate models are costly to run: it can take several months to simulate a few hundred model years (Dringnei et al., 2008). This limits the collection of existing simulations. If the existing data is deemed insufficient to design a robust emulator, it is expensive to significantly increase the available data. An example of this is Castruccio et al. (2014). They recognized the need to run more scenarios of the Community Climate System Model, version 3 (CCSM3; Collins et al. 2006; Yeager et al. 2006). In choosing new scenarios, they did not follow any experimental design procedure, and computational costs limited them to five new runs.

The computational cost of running complex climate models makes it imperative that input scenarios are chosen to maximize the information gained from these computations. There is no reason to believe that conventional RCPs will be efficient input scenarios for the purpose of constructing emulators. This paper takes a mathematical view motivated by approximation theory. Intuitively, the input scenarios should be orthogonal in some sense. We first apply Principal component analysis (PCA) to four conventional RCPs and find that they jointly contain little more information than one scenario would. We then examine the effectiveness of four scenarios constructed from orthogonal polynomials. The orthogonal polynomial scenarios do not look like anything we expect will happen but that is not important here. As intuition from approximation theory would indicate, we find that our four orthogonal polynomial input scenarios produce a significantly better emulator than the four standard RCPs.

Our emulator can be directly used to improve IAMs. Currently, most IAMs are deterministic, assuming that future climate and economy are perfectly predicable. Any model that ignores real-world economic and climate-related uncertainties misses much, if not most of the real uncertainty that policy makers have to face. By contrast, stochastic IAM formulations account for random variability of economic development and risks arising from lacking perfect knowledge about the climate system.

IAMs, deterministic or stochastic, should use as many state variables as required to ensure a realistic specification of the climate. One commonly used climate model is MAGICC, a reduced-complexity climate emulator (Meinshausen et al., 2011a). Since the computational complexity of solving dynamic models increases with their dimensionality a dynamic system of the size of MAGICC is too large to be commonly applied in advanced economic frameworks.

Even studies that account for some kind of uncertainty often point to the "curse of dimensionality" as an excuse for their simplified representations of the climate (EPA, 2010; Webster et al., 2012; Newbold et al., 2013; Jensen and Traeger, 2014). Recent advances in computational methods have described ways to solve high-dimensional economic models, even beyond 100 dimensions (Maliar and Maliar, 2015; Judd et al., 2011; Cai et al., 2015; Brumm and Scheidegger, 2016). Combining our approach to the construction of emulators with these new computational methods offers the potential to build more realistic models.

We test the usefulness of our emulation approach for stochastic IAMs by evaluating its ability to accurately simulate the distribution of temperatures in response to a stochastic emission process. We find that it does a very good job at this task.

The remainder of the paper is organized as follows. In Section 2 we describe the methods used for approximation; we motivate the use of orthogonal scenarios, explain the procedure of constructing them, and present the general model we estimate. Based on the results of the estimations, Section 3 states the specifications for recommended representations of the climate system and assesses the performance on alternative scenarios and under different sets of initial parameters of the climate model. Section 4 concludes.

2 Approximating high-dimensional models

The statistical approach to building emulators is a key part of much of IAM analysis. The critical issue is a thoughtful selection of scenarios for this purpose. In this section, we first examine the statistical character of the conventional approach using RCPs and then describe and evaluate our orthogonal polynomial approach. Before doing so, we briefly describe our source for temperature predictions, MAGICC.

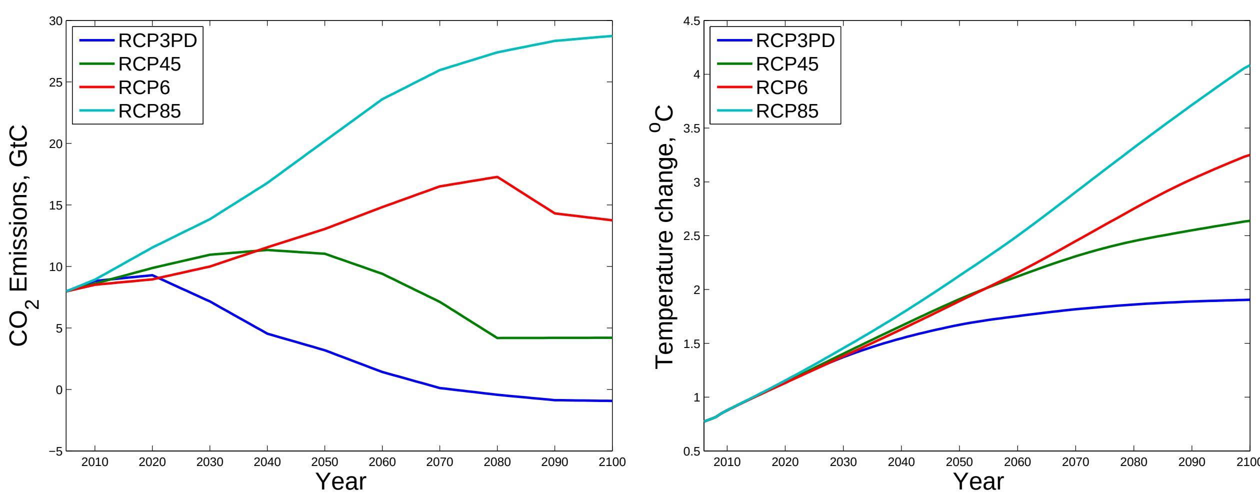
2.1 MAGICC

MAGICC is a carbon cycle-climate model to emulate the insights from large and complex AOGCMs. The emulation task is to generate global temperature responses (output variable) to various exogenous emission scenarios (input variable). In the climate research community, MAGICC is considered as a reduced complexity climate model. Yet, it includes representations of the most essential physical and biological components of complex AOGCMs. Despite being a "simple" model, MAGICC performs exceptionally well in emulating the results of the large AOGCMs, see Meinshausen et al. (2011a). Therefore, MAGICC has been used in the recent IPCC reports (IPCC, 2014) as the prime tool for evaluating carbon and climate responses to various emission scenarios. MAGICC is publicly available and easy to operate. According to Meinshausen et al. (2011a), MAGICC is flexible enough to deliver accurate results when running scenarios outside of the original calibration space. For the purpose of this paper, MAGICC is, therefore, best suited to generate reliable responses of global temperature to any emission scenario—responses that we use as benchmarks for evaluating the accuracy of our statistical method.

2.2 Conventional emission scenarios

Four basic RCPs endorsed by the IPCC are the most common and ready-to-use scenarios that currently serve as a common base for integrated climate/economic modeling and model inter-comparison projects (Taylor et al., 2012). Each RCP data set specifies the concentration and emission levels of greenhouse gases and other forcing agents. Fig. 1 displays CO_2 emissions paths specified by the RCPs and the corresponding temperature rise as predicted by MAGICC.

The statistical approach to emulator construction uses these emission–temperature pairs to estimate a single time series model where temperature is the dependent variable and lagged dependent variables and independent variables are on the right-hand side. The structure is similar to a pooled cross-section problem. Even though we use terms like "statistical approach" it needs to be emphasized that there is no underlying stochastic structure to the problem. The problem is really one of approximation where we want to find a simple dynamic model relating temperature to emissions with small prediction errors.



(a) CO_2 emissions, RCP scenarios.

(b) Temperature anomaly, RCP scenarios.

Fig. 1. RCP emissions scenarios (a) and corresponding predictions of temperature (b)

PCA indicates that the RCPs are not likely to be a good set for the purpose of estimating an emulator. Fig. 2 displays the variance decomposition implied by PCA, and shows that the first principal component carries more than 94 percent of the total variance in the set. This indicates that the first principal component, which is a linear combination of the four RCPs, provides as much information about emissions as the four scenarios do collectively. This makes it unlikely that the four RCPs are an efficient choice for estimating an emulator.

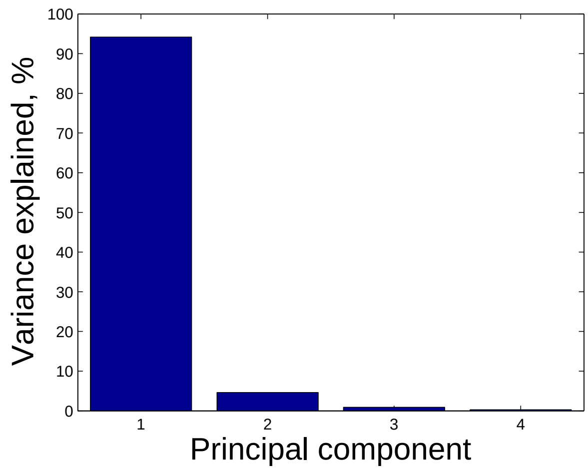


Fig. 2. Principal component analysis of RCP scenarios

2.3 Orthogonal emission scenarios

An efficient collection of input scenarios would make each one different from the others; more formally, they should be orthogonal to one another in some sense. Therefore, we use Chebyshev polynomials as our input scenarios. Chebyshev polynomials of degree n have the general form

$$F_n(x) = \cos(n \arccos(x)). \tag{1}$$

They are orthogonal with respect to the weighting function $1/\sqrt{1-x^2}$. Each nth degree Chebyshev polynomial has n roots (also called optimal nodes) given by

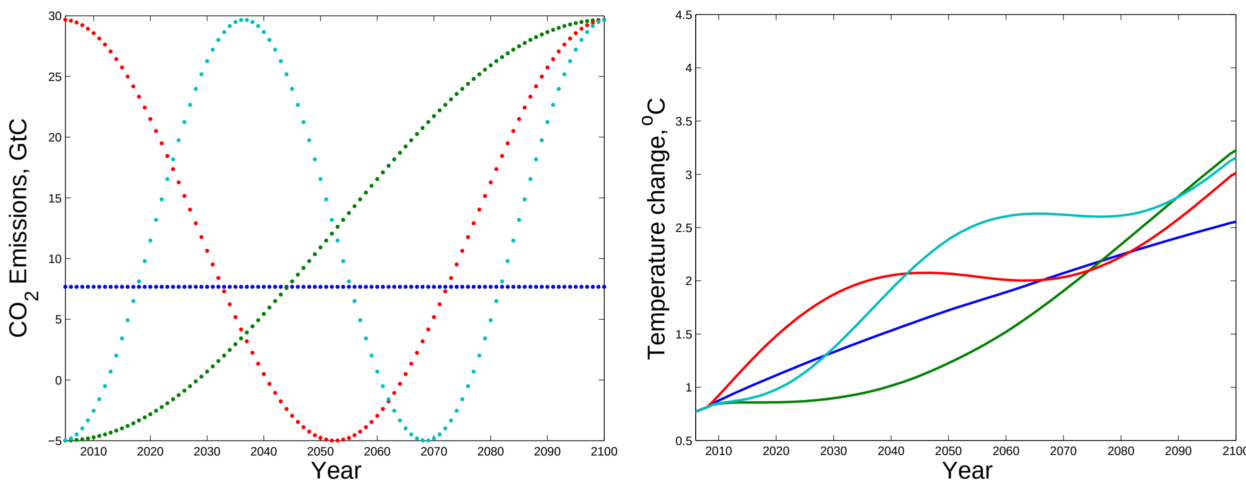
$$x_k = \cos\left(\frac{\pi}{2}\frac{2k-1}{n}\right). \tag{2}$$

An important property for our study is that the polynomials' values taken on N optimal nodes satisfy the following discrete orthogonality condition:

$$\sum_{k=0}^{N-1} F_i(x_k)F_j(x_k) = \begin{cases} 0 & : i \neq j, \\ N & : i = j = 0, \\ N/2 & : i = j \neq 0. \end{cases} \tag{3}$$

To design the input scenarios for n years we first take the values of the Chebyshev polynomials of degrees 0 to m on n optimal nodes. The discrete orthogonality property ensures that the resulting strings of values are uncorrelated. Next, we scale these values to the range close to the range of CO_2 emissions in the RCP scenarios. The zero-degree polynomial is an exception: it corresponds to a scenario of constant annual emissions, or the steady state of the economy, and is therefore set to the last historical value of CO_2 emissions.

The resulting values represent CO_2 emissions levels in n years—in our case 2005–2100 (Fig. 3a). Here and in all other scenarios used in this study the annual emissions of all other gases over this period are set to their average levels across the four RCP scenarios. The emissions of all gases for all years prior to 2005 are kept at their historical values. We set m to 3 so that using the four designed scenarios can be compared with using the four RCPs.



(a) CO_2 emissions, orthogonal scenarios.

(b) Temperature anomaly, orthogonal scenarios.

Fig. 3. Orthogonal emissions scenarios (a) and corresponding predictions of temperature (b).

The degree-zero polynomial (a constant) would generally not be used as an RCP because it is not "interesting" (Fig. 4), however we include it because its inclusion is required by approximation theory. With the constant function as a scenario, we can construct an orthogonal Chebyshev approximation that will span a vector space of plausible paths, and has the form

$$E_{RCP_i}(t) \approx \sum_{k=0}^{m} a_k F_k(t), \tag{4}$$

where $E_{RCP_i}(t)$ is the level of CO_2 emissions in the i-th RCP scenario in year t, $F_k(t)$ is the scaled value of Chebyshev polynomial of degree k at node t, and a_k is the weight corresponding to the polynomial of degree k.

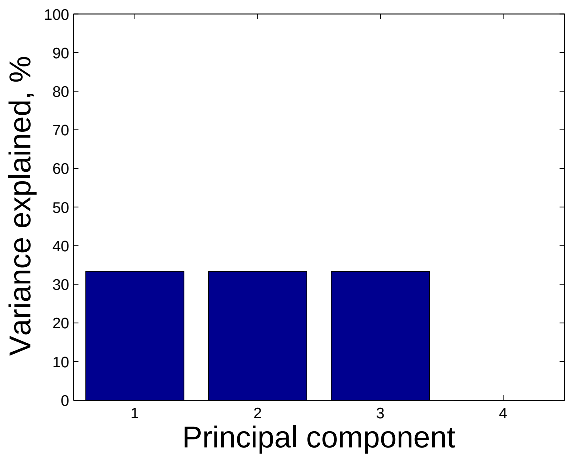
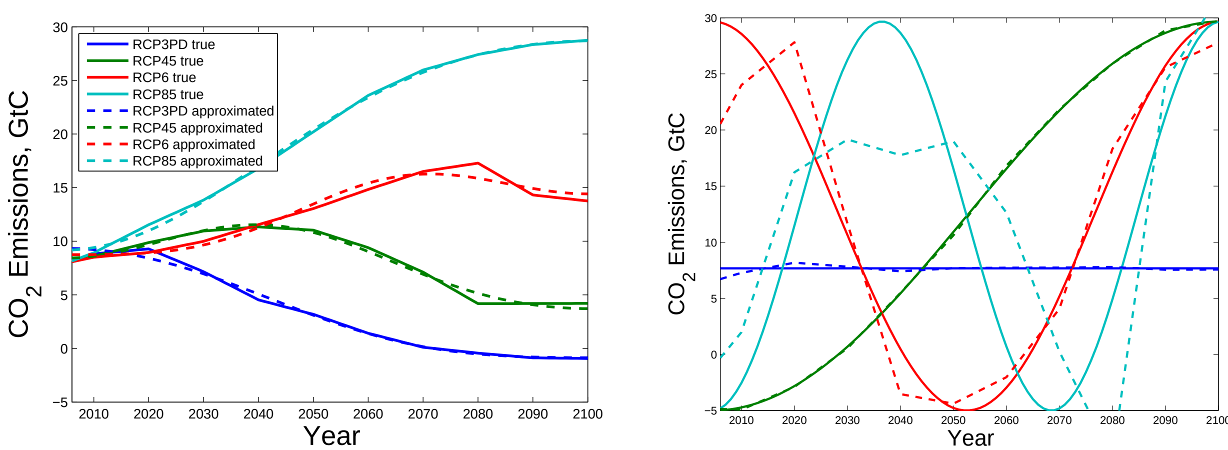


Fig. 4. Principal component analysis of orthogonal scenarios.

It is obvious that future emission paths will not look anything like the degree two or degree three Chebyshev polynomials, but the task here is not to discuss what the likely paths are. The task is to find a collection of emission scenarios which will allow us to extract as much information as possible from a mathematical model of the climate. However, the Chebyshev paths should be able to produce good approximations of the standard RCP scenarios because they are of particular interest in the literature. Fig. 5a shows that our Chebyshev polynomials do a good job; in fact, the root mean squared error (RMSE) is only 0.36 gigatons of carbon per year ($GtCyr^{-1}$).



(a) Approximation of RCP scenarios.

(b) Approximation of orthogonal scenarios.

Fig. 5. Approximation of RCP scenarios with orthogonal scenarios (a) and approximation of orthogonal scenarios with RCP scenarios (b).

We next check our basic argument that the standard RCPs do not form a good collection for emulator construction. Our test is to see how well the standard RCPs do in approximating the Chebyshev scenarios. Fig. 5b shows that they do not do well, and produce approximations with an RMSE of $2.55 GtCyr^{-1}$. This result is consistent with the PCA analysis that says the four standard RCPs contain little more information than the best one on its own.

2.4 General estimation method

We use the following general form of a dynamic linear model:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \sum_{j=2}^{J} \beta_j X_{j,t-1} + \varepsilon_t, \tag{5}$$

where Y_t is the predicted variable in year t, $X_{j,t}$ is the jth covariate in year t, and J is the number of covariates in the model. To capture the residual autocorrelation, we assume that regression errors follow an ARMA(1,1) process and its residuals are normally distributed:

$$\varepsilon_t = a\varepsilon_{t-1} + bu_{t-1} + u_t, \ u_t \overset{iid}{\sim} N(0, \sigma^2). \tag{6}$$

To generate the data for training the model, we run the designed orthogonal emissions scenarios in MAGICC. The resulting temperature values are the benchmark values of the predicted variable. The potential covariates are taken from the corresponding input data, e.g., CO_2 emissions, concentrations or cumulative emissions. For each potential set of covariates we pool the data of the four scenarios together and estimate the parameters using a maximum likelihood estimator.

To test our model, we take the CO_2 emissions scenarios given by the four RCPs. We use the average RMSE to assess the corresponding predictions.

3 Results

We have constructed several two- and three-dimensional prediction models and present them in this section.

3.1 Best-performing low-dimensional model

We find that the following model specification with the temperature anomaly T_t (deviation from the preindustrial level) in year t and cumulative CO_2 emissions C_t as an exogenous variable produces the best predictions:

$$T_t = \beta_0 + \beta_1 T_{t-1} + \beta_2 C_{t-1} + \varepsilon_t,$$
$$\varepsilon_t = a\varepsilon_{t-1} + bu_{t-1} + u_t,\ u_t \overset{iid}{\sim} N(0, \sigma^2). \tag{7}$$

Here cumulative emissions are accumulated from 1765 to year t and measured in $GtC \cdot 10^{-3}$. The first row of Table 1 reports the estimated values for the parameters of the model.

Table 1

Approximation results for different model specifications.

Model	β_0	β_1	β_2	β_3	a	b	σ	RMSE
(7)	0.2500	0.7650	0.3632		0.9805	0.2128	0.0022	0.0338
(8)	0.1188	0.6874	0.1503		0.9878	0.1854	0.0018	0.0426
(9)	0.1230	0.6820	0.0286	0.1445	0.9873	0.1832	0.0018	0.0411

RMSE is the average error of prediction for the testing set of four RCP scenarios.

As shown in Fig. 6, this model produces very accurate predictions: the average RMSE across all four testing scenarios is only about 0.03℃. Thus, we show that temperature levels can be inferred immediately from CO_2 emissions data within a one-line model that performs well on the conventional scenarios. Our results therefore are consistent with recent studies that suggest a linear-proportional relationship between global warming and the level of cumulative CO_2 emissions (Allen et al., 2009; Zickfeld et al., 2009; Matthews et al., 2009; IPCC, 2013).

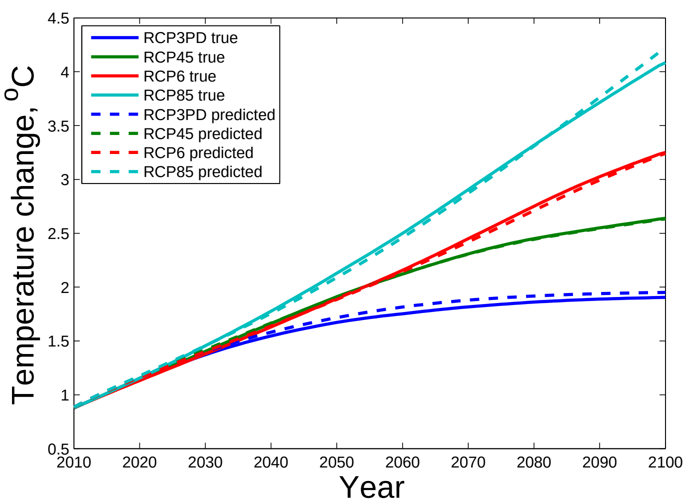


Fig. 6. Out-of-sample temperature predictions of the best fitting model (7).

The required number of scenarios becomes very important when complex, computationally expensive models are emulated. Our approach suggests a clear way to determine the number of runs that such an exercise requires. Fig. 7 demonstrates that only up to three runs contribute to emulation precision significantly, after which the prediction errors level off. The key to successful emulation in this case is the efficient design of the input data set before the complex climate models are run.

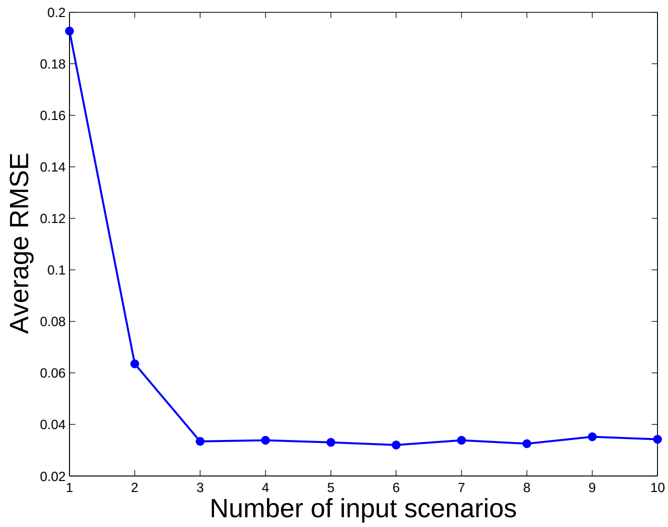


Fig. 7. Average out-of-sample prediction error as a function of the number of orthogonal scenarios used as input.

Furthermore, our recommended prediction model can be embedded within a dynamic system of equations, such as often employed in integrated assessment models. Many IAMs—in particular those focusing on intrinsic uncertainty in social decision–making, suffer from poor representations of the climate system due to computational constraints. Given the accuracy of our predictions, integrated assessment modellers could include a simple yet accurate low-dimensional mapping of emissions to temperature levels into their models.

Given the structure of the two dynamic equations from above, $T_t = f(T_{t-1}, C_{t-1}, \epsilon_t)$ and $\epsilon_t = g(\epsilon_{t-1}, u_{t-1}, u_t)$ an economic IAM would also require adding a dynamic equation for cumulative emissions, $C_t = h(C_{t-1}, E_t)$, where E_t is some emission scenario resulting from the model's optimization framework.

3.2 Alternative specifications

Here, we present alternative functional forms of 2-dimensional and 3-dimensional representations of the climate system. The underlying objective of obtaining these representations is to fit the temperature paths of the four RCP scenarios.

The alternative 2-dimensional model includes CO_2 concentrations S_t measured in $ppm10^{-2}$ in year t as an exogenous covariate,

$$
\begin{aligned}
&T_t = \beta_0 + \beta_1 T_{t-1} + \beta_2 S_{t-1} + \varepsilon_t \ , \\
&\varepsilon_t = a\varepsilon_{t-1} + b u_{t-1} + u_t, \ u_t \overset{iid}{\sim} N(0, \sigma^2).
\end{aligned} \tag{8}
$$

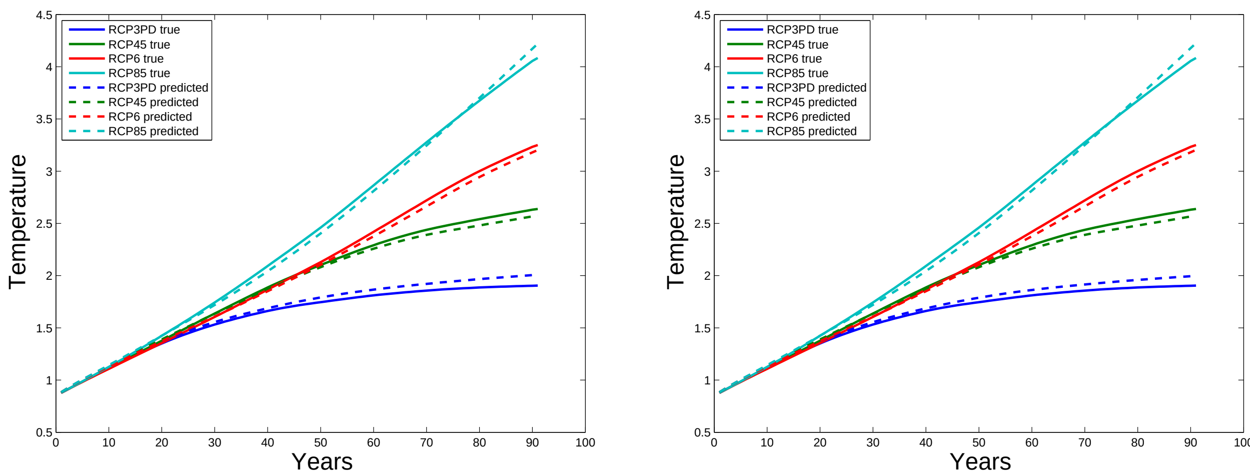
As shown in Table 1, cumulative CO_2 emissions are a better predictor; however, if only CO_2 concentrations are available, the resulting emulator would also generate good predictions.

The suggested 3-dimensional model includes both CO_2 concentrations and cumulative emissions,

$$
\begin{aligned}
&T_t = \beta_0 + \beta_1 T_{t-1} + \beta_2 S_{t-1} + \beta_3 C_{t-1} + \varepsilon_t \ , \\
&\varepsilon_t = a\varepsilon_{t-1} + b u_{t-1} + u_t, \ u_t \overset{iid}{\sim} N(0, \sigma^2).
\end{aligned} \tag{9}
$$

However, Table 1 indicates that the extended model does not outperform the lower-dimensional one.

Fig. 8 shows the out-of-sample predictions of the temperature levels produced by the two alternative models.



(a) 2-dimensional model (8).

(b) 3-dimensional model (9).

Fig. 8. Out-of-sample temperature predictions of the alternative models.

3.3 Performance verification

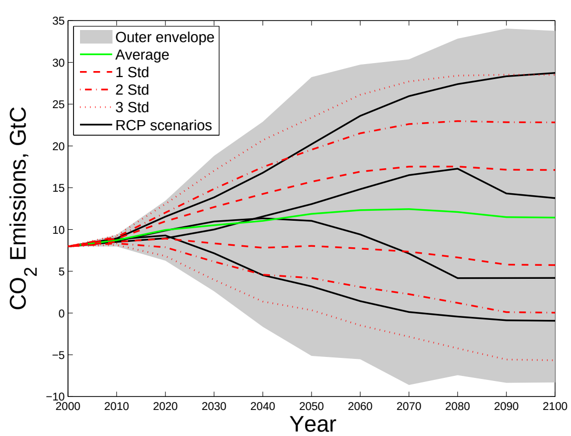
Advances in climate and economic research bring new knowledge about the likely paths of socioeconomic development and the estimated climate impacts. An ongoing process of scenarios creation addresses the need of the research community in new scenarios consistent with current understanding of possible global developments and reflecting the associated uncertainty (Moss et al., 2010). The most recent example is the Shared Socioeconomic Pathways which complement the existing scenarios with alternative paths of socioeconomic development (Riahi et al., 2016; O'Neill et al., 2014).

As new generations of scenarios get incorporated into integrated assessment modeling, they become a new common base for scientific research in this field. However, models trained on traditionally used scenarios, such as RCPs, might perform poorly with the new ones coming into play.

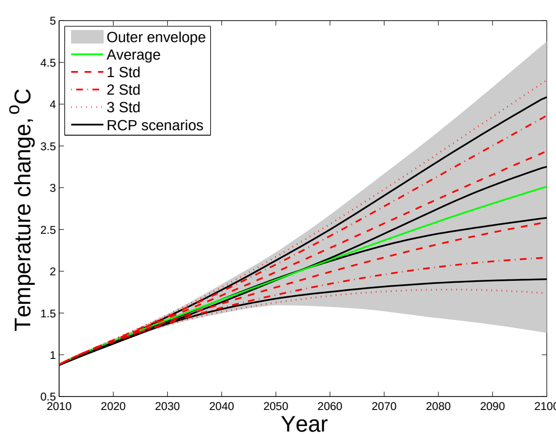
Our additional task is to ensure that the proposed emulator suits any arbitrary scenarios within the range considered plausible in the literature. Since RCPs were created to represent a wide range of scenarios in the preceding literature, we would like to assess the performance of our model on the scenarios that RCPs might represent. In particular, we construct a stochastic process that allows us to generate any number of RCP-like scenarios. We then verify if the temperatures levels predicted for these scenarios are close to the predictions produced by MAGICC. In principle, the following stochastic process could generate the CO_2 emission paths of the four RCP scenarios:

$$E_t = E_{t-1} + \varepsilon_t,\ \varepsilon_t \sim N(\mu_t, \sigma_t), \tag{10}$$

where E_t is CO_2 emissions in year t and μ_t and σ_t are estimated from the four RCP scenarios for each year t.

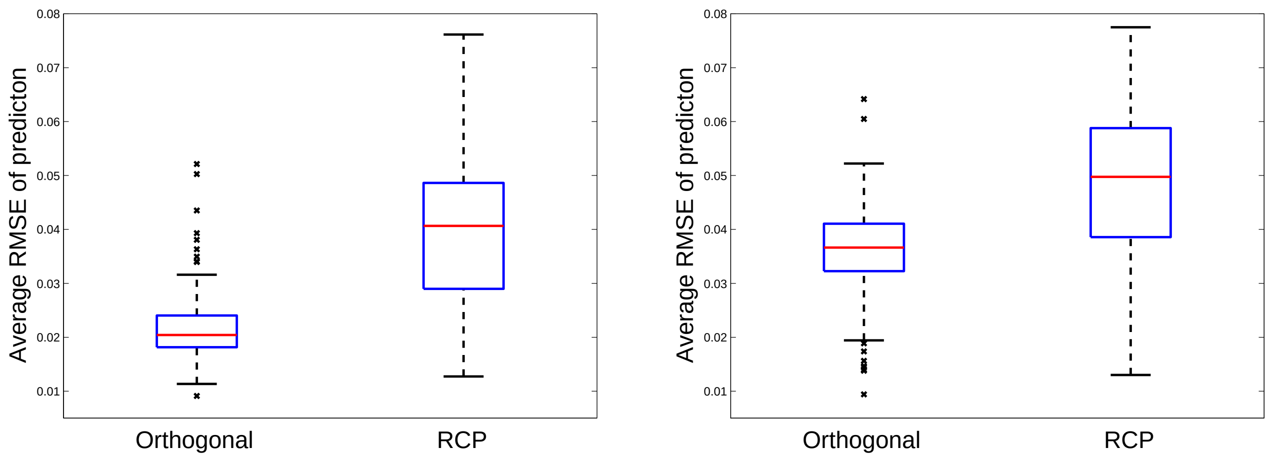


(a) CO_2 emissions scenarios.



(b) Temperature anomaly.

Fig. 9. Statistical distribution of the simulated scenarios and corresponding temperature predictions (data generated using four orthogonal scenarios).



(a) Model with cumulative CO_2 emissions. (b) Model with CO_2 concentrations.

Fig. 10. Out-of-sample prediction errors of two of the proposed models for simulated scenarios.

We generate a sample testing set of 10,000 realizations of the stochastic process (Fig. 9) and compute the model's prediction errors for each type of scenarios. In the case of using orthogonal scenarios as an input for MAGICC and the best fitting model as an emulator, we obtain an average RMSE of only 0.02℃. Fig. 10 compares the performance of orthogonal scenarios with that of the RCP scenarios as input sets for initial model runs. The average error of prediction across the simulated paths reduces significantly when the designed input paths are orthogonal.

3.4 Performance on alternative model settings

Model uncertainty strongly affects predictions of the temperature response to emission paths. The predictions of the complex climate models are known to diverge greatly from one another—even when calibrated to the same initial conditions and run with the same forcing scenarios they span a wide range of possible system forecasts. Model uncertainty might stem from differences in assumptions, modeled components, and the structure of those components (Tebaldi and Knutti, 2007). These differences can have a significant impact on the accuracy of emulating models with regard to approximating the outcomes of complex climate models (Meinshausen et al., 2011a).

So far, we have used in our analysis only one (default) combination of the 20 atmosphere-ocean general circulation models (AOGCMs) and 10 carbon cycle models emulated in MAGICC (Meinshausen et al., 2011b). However, as documented by numerous intercomparison projects, there is great variability among complex models in terms of their predictions of climate response to emission scenarios (Taylor et al., 2012; Gillingham et al., 2015).

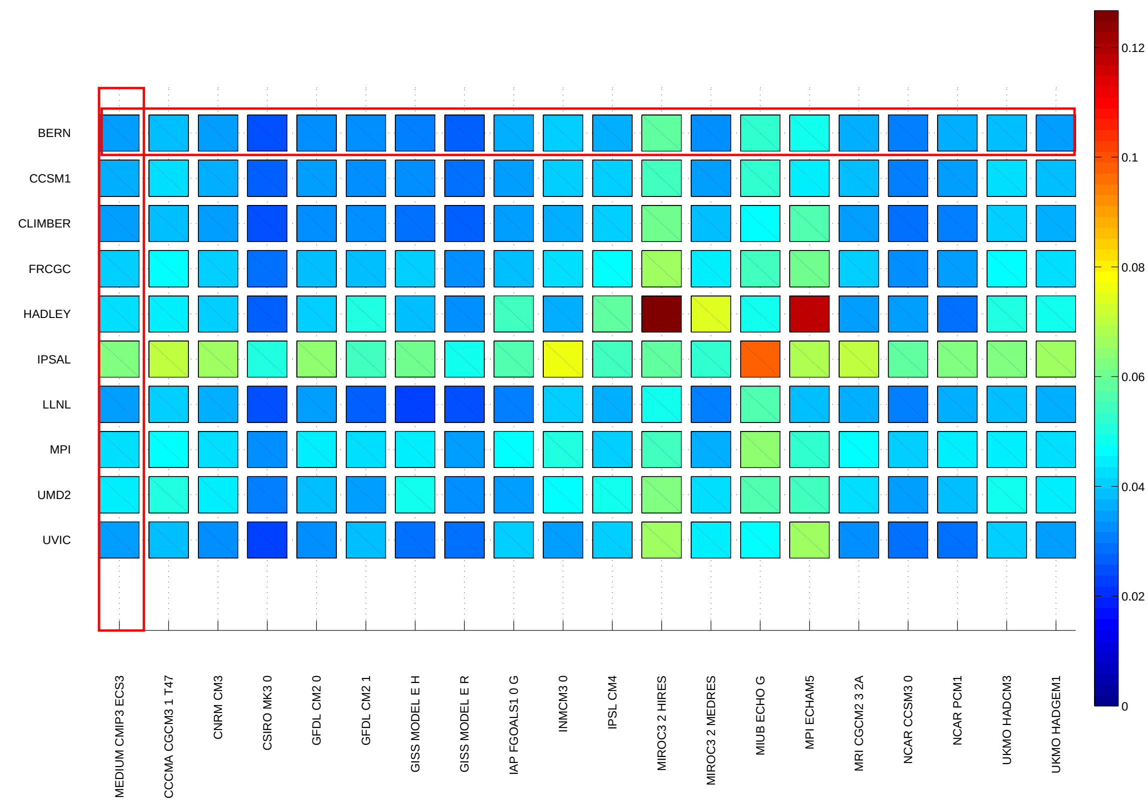


Fig. 11. Model uncertainty: RMSEs for combinations of 20 AOGCM and 10 C4MIP. The settings used in MAGICC as default values are marked with red rectangles

Therefore we would like to ensure that our approach is not restricted to a single climate model, but may be applied to any of the existing ones. To do so, we additionally check whether the method performs equally well for the 200 different sets of parameters obtained from combining each of the 20 AOGCM settings with the each of the 10 carbon cycle settings emulated with MAGICC (Fig. 11).

We find that the average RMSE across all settings is only about 0.04℃, and in most of the 200 model combinations in MAGICC the average RMSE does not exceed 0.07℃. In general, we find that all combinations produce low approximation errors[2]. The insights from Fig. 8 could be useful for improving emulation exercises in the future, and for estimating model uncertainty. Overall, we conclude that our emulating technique and the recommended low-dimensional model perform very well on the plausible settings of the underlying model parameters.

4 Conclusion

New emission scenarios and socioeconomic pathways are constructed on a regular basis. As the work on the next IPCC Assessment Report has commenced, emulation of large and complex climate models will certainly be on the research agenda. The known resource limitations of running large climate models call for efficient emulation techniques. We recommend the use of orthogonal emission scenarios for an efficient yet accurate approximation of climate models. The orthogonal scenarios based on Chebyshev polynomials display quite unrealistic emission paths. However, the purpose of using orthogonal scenarios is purely technical—namely to extract as much information as possible from the complex model.

Using the global temperature anomaly as a predicted response variable, we produce an econometric model—a low-dimensional system of mapping emissions to temperature levels for the 21st century. Our simulations confirm that the model performs well on conventional scenarios; the precision of approximation stays high under various settings of climate and carbon cycle parameters. The designed system of equations can be directly implemented in dynamic stochastic general equilibrium models often used in macroeconomics, allowing one to study optimal policies for dealing with global warming under conditions of uncertainty in terms of social decision-making.

[2]There are some notable differences among the individual climate and carbon cycle models, such as "Hadley" and "IPSAL", which are both known for strong carbon cycle feedbacks.

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